

15. (a) Prove that the characteristics X of elements of a population has the Cauchy distribution given by the density $f(x) = \frac{1}{\pi[1 + (x - Q)^2]}$, where Q is an unknown parameter.

Or

- (b) Let V be an unbiased estimate of the parameter Q , and let U be a sufficient estimate of Q . Then prove that the random variable $E(V/u)$ is an unbiased estimate of Q . If, moreover, the variance $D^2(V)$ exists, the inequality $D^2[E(V/u)] \leq D^2(V)$ holds.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

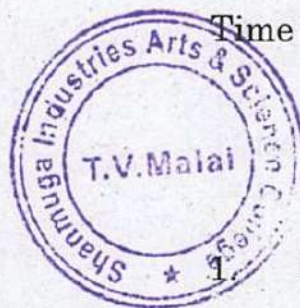
16. Brief note on the X^2 distribution.
17. Derive Fisher's Z distribution.
18. State and prove Smirnov's theorem.
19. Derive the Unbiased estimates.
20. State and derive Rao-Cramer inequality.

APRIL/MAY 2025

**23PEMA24B — MATHEMATICAL
STATISTICS**

Time : Three hours

Maximum : 75 marks



SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define Static.
2. Write the Static χ^2 distribution
3. Define Pearson's χ^2 statistic.
4. Define non-centrality parameter.
5. Define Statistical Hypothesis.
6. Write about parametric test.
7. Define Contingency table.
8. Discuss about consistent estimates.
9. Define unbiased estimate.
10. What is known as bias of the estimate?

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions.

11. (a) Derive Student's t distribution.

Or

- (b) Prove that the sequence $\{F_n(t)\}$ of distribution function of *Student's t* with n degrees of freedom satisfies for every t the relation $\lim_{n \rightarrow \infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-t^2/2} dt$.

12. (a) Suppose that have a simple sample of $n = 50$ elements drawn from a population in which the characteristic X has a distribution with the standard deviation $\sigma = 5$ and unknown expected value m . From the sample we have obtained $\bar{x} = 2$ Test the hypothesis ($H_0 = 0$).

Or

- (b) Discuss Independence test by using Contingency table.
13. (a) The characteristic X of a population has the normal distribution $N(m, \sigma)$ with σ unknown and m known. As an estimate of this parameter, we take the mean deviation of simple samples of size n multiplied by $\sqrt{0.5\pi}$, that is, the statistic
$$U = \frac{1}{\pi} \sqrt{0.5\pi} \sum_{k=1}^n |X_k - m|.$$

Or

- (b) The characteristic X of elements of a certain population has the normal distribution $N(m, \sigma)$, where the expected value m is known but the standard deviation σ is not known. If the static S_0^2 is unbiased estimator of σ^2 , Show that S_0 is not unbiased estimate of σ .

14. (a) Consider the class D of distribution functions of random variables X whose second moments exist. Let the variance σ^2 of X be unknown. As an estimate of this parameter, we take the statistic
$$S^2 = \frac{1}{n} \sum_{k=1}^n X_k^2 - \overline{X^2},$$
 where the random variables X_k are independent and have the same distribution as X . Where the statement follows unbiased estimate if explain in brief?

Or

- (b) Define unbiased estimates and derive example.

